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Relativistic Quantum Logic

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On the basis of the well-known quantum logic and quantum probability a formal language of relativistic quantum physics is developed. This language incorporates quantum logical as well as relativistic restrictions. It is shown that relativity imposes serious restrictions on the validity regions of propositions in space-time. By an additional postulate this relativistic quantum logic can be made consistent. The results of this paper are derived exclusively within the formal quantum language; they are, however, in accordance with well-known facts of relativistic quantum physics in Hilbert space.

1. INTRODUCTION

In recent years a formal language of quantum physics has been developed, which may be considered as the object language with respect to a single quantum physical system and its properties. This language, often called quantum logic on account of the formal logic contained in it, is the most abstract schema of quantum physics which does not make use of algebraic and analytical techniques (Mittelstaedt, 1978; Stachow, 1981a). Algebraic representations of this quantum language are given by a lattice structure and a Baer*-semigroup, respectively (Stachow, 1981a; 1978d), which are well known from earlier algebraic investigations of quantum mechanics (Birkhoff and von Neumann, 1936; Foulis, 1960; Jauch, 1968). The realization of these abstract structures in Hilbert space lead to the usual formalism of nonrelativistic quantum theory (Stachow, 1981a; Jauch, 1968; Piron 1976). In this language a probability theory has been developed, which is based on the same single case semantics as the language and is thus also related to an individual system (Stachow, 1978b; 1979; 1981b). The algebraic representation of this quantum probability calculus leads to the well-known quantum probability theory (Jauch, 1974; Mittelstaedt, 1976a). The quantum language and its extension by the quantum probability

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calculus allow for a realistic description of a quantum system, its preparation, its properties, and its probabilities.

In the present paper this quantum language (Q language) and quantum probability is generalized for the case of relativistic space-time. This formal language of relativistic quantum physics is obtained here by the additional restrictive postulate, that the quantum physical observer operates only in local regions of space-time and uses only local concepts for the construction of his language (Section 2). In this way all assumptions can be avoided which would lead to the Newtonian space-time. Moreover, the local point of view makes it possible to develop the Q language in the framework of the Minkowskian space-time as well as in Riemannian space-time. In this way one could presumably perform a project which was first formulated by Finkelstein: The transition from classical logic to quantum logic should be replaced by transition to a less rigid structure which depends on the respective metric in space-time (Finkelstein, 1969). Here, however, we will restrict our investigations to the homogeneous space-time of special relativity.

In Q language the accumulation of knowledge about a system is described such that a certain conception of measuring processes is tacitly used. In Section 4 this measuring process is made explicit exclusively in terms of Q language and without any recourse to Hilbert space quantum physics. For commensurable propositions an important theorem can be proved which in Hilbert space quantum physics corresponds to a well-known theorem due to Lüders (1951) and Schlieder (1968). Here, however, the theorem is proved exclusively by means of Q language and quantum probability.

On the basis of this result the restrictions can be formulated which must be imposed on Q language in relativistic space-time. Here the validity of propositions proved by measurements is not only restricted to some time intervals (as in Newtonian space-time) but restricted to certain validity regions in space-time. Furthermore, the limitations of signals due to the light-cone structure of space-time become important if propositions are proved at points with spacelike distance. Again it should be emphasized that all these restrictions of the relativistic Q language are derived here exclusively within the framework of this language. In terms of Hilbert space quantum physics, analogous investigations concerning states and operators have been carried out recently by Schlieder (1968). However, since the results of the present paper had to be obtained without any recourse to Hilbert space mathematics, the investigations mentioned were only able to serve as guide lines.

It is not obvious that the relativistic quantum language which is obtained in this way by imposing several restrictions on the well-established

Q language is not self-contradictory. In fact, it is shown here (Section 6) that an additional consistency postulate (C postulate) must be demanded, if the following requirements are to be fulfilled: The realistic interpretation of the history of a single system should be compatible with the relativistic Q language, and the quantum physical long-range correlations should not violate relativistic causality. These results, which are summarized in the LC theorem (Section 6), again correspond to well-known requirements and theorems of Hilbert-space quantum physics, e.g., the C postulate corresponds to the locality condition of quantum field theory (cf., e.g., Streater and Wightman, 1964). However, it seems to be noteworthy, that all results of the present paper could be derived within the abstract framework of relativistic quantum language, and are thus generally relevant irrespective of the particular mathematical formulation of the physical theory.

2. THE OBSERVER IN QUANTUM PHYSICS AND RELATIVITY

It is the goal of this paper to develop the basic concepts of a formal language of relativistic quantum physics. If the various terms of this language are introduced operationally, the physical conditions under which measuring results can be obtained become important. These conditions can adequately be expressed as restrictions which must be imposed on the *observer*. In this sense it has been emphasized by *Einstein* for the domain of relativistic physics, and by *Bohr* for the domain of quantum physics, that the observer is an inevitable element of any description of nature. In the present paper the observer is considered at the same time as *speaker* of the scientific language of physics, whose possibilities of investigating properties of a physical system belong to the preconditions of the respective language. Hence in order to construct a language for quantum physics and relativity we first consider separately the quantum physical observer and the relativistic observer and try to combine the possibilities of these two observers by introducing the *universal* observer. This universal observer will then be the speaker of the language of relativistic quantum physics, which is investigated in the following sections of this paper.

The Quantum Physical Observer. We consider a quantum physical system S, i.e., an elementary particle, a nucleus, an atom, etc. The preparation of the system, its contingent properties, and the temporal sequence of measurements are described by a *quantum physical observer* B_Q. According to *Bohr* it is essential that this observer—considered as a physical object—has macroscopic dimensions. He is equipped with macroscopic instruments for the preparation of S and for the measurement of its contingent properties. For the description of the preparation, the history, and the present state of the system S in question, the observer B_Q makes use of a language $\mathfrak{S}_Q$ which will be called the *language of quantum physics* (Mittelstaedt, 1978; Stachow, 1981a). Since the mutual commensurability of two arbitrary properties of S cannot generally be guaranteed, in the language the simultaneous decidability of two propositions is not generally presupposed but subject to a testing procedure which must be performed in every single case.

The elementary propositions of the language $\mathfrak{S}_Q$ are of the form $a(S, t')$ and state that the system S has at the time t' a property E_a. The proof of this statement has to be performed by a quantum physical measuring process which makes use of a macroscopic measuring device. It is assumed in this description that the observer B_Q and its measuring apparatus operate in a finite region R of space-time. However, it is also assumed that the proposition $a(S, t')$ gives a correct description of S for all points (x^k, t) of space-time with $t \geqslant t'$. Consequently a sequence $a(S, t')$, $b(S, t'')$, $c(S, t''')$ with $t' < t'' < t'''$ describes the history of S for space-time points (x^k, t) with $t' \leqslant t < t''$, $t'' \leqslant t < t'''$, $t''' \leqslant t$, respectively. Hence by means of the language $\mathfrak{S}_Q$ the observer B_Q describes the state and the history of S within the framework of the nonrelativistic, Newtonian space-time.

The Relativistic Observer. We consider a macroscopic physical system Σ in the sense of classical relativistic physics, i.e., a macroscopic body, an electromagnetic field, etc. A system of this kind will be observed and described by an observer B_R, who operates in a finite region R of space-time and who is equipped with a material inertial basis, with clocks and measuring rods. This *relativistic observer* B_R makes use of space-time coordinates $K_B(x, t)$, which are at rest with respect to the material reference basis of B_R and which are chosen such that force-free test bodies of a given ensemble $\Gamma = \{\Gamma_i\}_i$ move uniformly on straight lines. Statements of the observer B_R about the properties of the physical system Σ at a certain space-time point (x, t) are thus dependent on the respective coordinate system $K_B(x, t)$. Hence the particular system K_B of coordinates belongs to the constituents of the language $\mathfrak{S}_R^{(B)}$ of the relativistic observer B_R.

Different observers B_R, B'_R who are in relative motion to each other make use of different coordinate systems $K_B(x, t)$, $K_{B'}(x', t')$,... and of different languages $\mathfrak{S}_R^{(B)}$, $\mathfrak{S}_R^{(B')}$,.... The transformations between coordinate systems of this kind are given by special linear functions of the space-time coordinates. The requirement that not only the uniformity and rectilinearity of the motion of the elements Γ_i of Γ be preserved by this transformation, but also the temporal order of successive space-time events of Γ_i, leads to the generalized Lorentz transformation $\Lambda(v_\infty)$, which still contains an arbitrary constant v_∞. The Lorentz transformation $\Lambda(c)$ with $v_\infty = c$ is then obtained by the additional postulate that also the propagation of light has the same form for all observers (Levi-Leblond, 1976; Mittelstaedt, 1976/80; Pfarr, 1983).

Series of causally connected space-time events $\sigma\{\varepsilon_1, \varepsilon_2, \ldots\}$ are called *signals* (Mittelstaedt, 1976/80, p. 77). If the temporal order of two successive events $(\varepsilon_1, \varepsilon_2)$ is invariant under Lorentz transformations, these events have a timelike distance and thus the velocity between ε_1 and ε_2 is always $v(\varepsilon_1, \varepsilon_2) \leqslant c$. It is obvious that signals which consist of events of some Γ_i and light signals have this property. For other kinds of signals it does not follow from this derivation of the Lorentz transformation that they are propagated with $v_\sigma \leqslant c$. However it is generally accepted as a principle of relativistic physics that signals of any kind cannot proceed faster than light. This principle, which is often called *Einstein causality*, will be assumed to be valid for relativistic observers through this paper.

The Universal Observer. A quantum physical observer B_Q performs measurements with macroscopic instruments in a *macroscopic* region R of space-time. In his language $\mathcal{S}_Q$ he is subject to the restrictions which come from the possible mutual incommensurability of quantum physical propositions. Since the region R of space-time is not only macroscopic but also finite and *local* with respect to relativistic space-time dimensions, the observer B_Q may be considered at the same time as a relativistic observer B_R. As a relativistic observer he works with operationally defined coordinate systems $K_B(x^k, t)$ and is thus subject to the relativistic restrictions of space-time. In particular the exchange of information with other observers is restricted by the principle of *Einstein causality*, i.e., by the limited velocity of all signals.[1]

A quantum physical observer who at the same time is a relativistic observer will be called here a *universal observer* B_U. The universal observer is macroscopic from the quantum physical point of view and local in the sense of relativistic space-time. The region R of space-time in which his measurements are performed is large compared to atomic distances but small compared with light years, i.e., it has the dimensions of a laboratory. The universal observer is both subject to the quantum-mechanical and to the relativistic restrictions. He has to take into account possible incommensurabilities as well as the question as to whether the four-dimensional

[1]Here the principle of Einstein causality is—according to the previous section—considered as a restriction which refers to classical relativistic processes such as the propagation of light. Whether also quantum physical processes are restricted by this principle is for the present an open question, which will be investigated in Section 6 of this paper.

distance of two events is spacelike or timelike. The language of the universal observer will be called the language $\mathcal{S}_{RQ}$ of relativistic quantum physics.

In the present paper the space-time manifold will be assumed to be a Minkowskian space-time $\mathfrak{M}$ with the metric tensor $\eta_{\mu\nu} = \delta_{\mu\nu} \cdot \{+1, -1, -1, -1\}$. Hence we consider quantum physics within the framework of the space-time of special relativity. The present approach could, however, presumably be generalized also for the Riemannian space-time of general relativity, since all considerations of this paper are based on the *local* point of view, which deals with a local and inertial observer B_U, who uses a local Q language and a comoving local inertial coordinate system. Hence by convenient generalization of the formal derivation of this paper, gravitational fields could also be taken into account.

3. THE FORMAL LANGUAGE

In this section we briefly develop the formal language $\mathcal{S}_Q$ of quantum physics (Q language) which is used by a well-defined observer B_U. Here we are interested mainly in the formulation of a language of physics which takes into account the restrictions which come from the possible incommensurability of quantum physical propositions. The restrictions which must be imposed on the language on account of relativistic physics are considered here only as far as the local character of the language is emphasized and the observer B_U is considered as being located in a well-defined space-time region. The more important relativistic restrictions which come from space-time validity regions of propositions and from Einstein causality will be considered in Sections 5 and 6.

In order to constitute the object quantum language $\mathcal{S}_Q$ we begin with a quantum physical system S (atom, nucleus, elementary particle), the properties of which can be tested experimentally by the observer B_U. Propositions $A, B, \ldots$ about this system S which can be proved or disproved by measuring processes will be called *elementary propositions*. Since the truth or falsity of elementary propositions is determined by a well-defined proof procedure, these propositions are called *proof-definite*. Moreover we will assume that elementary propositions are *value-definite*, i.e., there always exists an experimental testing procedure which decides between truth and falsity of the respective propositions. According to the considerations of the previous section the testing processes are performed by B_U within a finite region R of space-time. Hence we consider only those elementary propositions which are proof-definite in the restricted sense that the proof processes are carried out in this *provability region* R of B_U. The set of these *local* elementary propositions is denoted here by $\mathcal{S}_e(R)$.

TABLE I.

Connective	Possibility of attack	Possibility of defence
$A \sqcap B$	1?, 2?	A, B
$A \wedge B$	1?, 2?	A, B
	$k(A, B)$?	$k(A, B)$!
$A \vee B$	?	A, B
		$\bar{k}(A, B)$
$A \rightarrow B$	A	B
	$k(A, B)$?	$k(A, B)$!
$\neg A$	A	

In quantum physics the simultaneous testability of two propositions $A, B \in \mathfrak{S}_e(R)$ is not guaranteed, except when the propositions A and B are commensurable. In order to incorporate the possibility of pairs of incommensurable propositions into the formal language, we introduce "availability propositions" $k(A, B)$ (commensurability) and $\bar{k}(A, b)$ (incommensurability) which can be tested by convenient series of measurements (Mittelstaedt, 1978; Stachow, 1981a). In particular $k(A, B)$ is considered to be true iff A and B can be tested in arbitrary sequence without thereby influencing the outcome of the respective trials. In the same way as elementary propositions, availability propositions also are tested by material processes within the local provability region R of B_U. Since elementary and availability propositions are tested by material processes in R, we will refer to these kinds of propositions as *local material propositions*. Starting from the set of material propositions we can further extend the scientific language by incorporating sequential and logical connectives. The connectives $A \sqcap B$ (A and then B), $A \wedge B$ (A and B), $A \vee B$ (A or B), $A \rightarrow B$ (if A then B) and $\neg A$ (not A) will be defined by the possibilities of attack and defence of these compound propositions within the material dialogue game D_m (Mittelstaedt, 1978; Stachow, 1981a). The attack and defence scheme reads as shown in Table I. In this table we have denoted by $k(A, B)$? the challenge to prove $k(A, B)$ and by $k(A, B)$! the successful proof of $k(A, B)$.

The meaning of this attack and defence scheme can best be illustrated by a proof tree (Stachow, 1980; 1981a), which represents the temporal sequence (from left to right) of successive experimental tests. (The temporal order corresponds to the rest frame of B_U). For $A \wedge B$, e.g., one obtains the proof tree shown in Figure 1.[2]

At each branching point, one of the material propositions A, B, $k(A, B)$ is tested. There is only one successful branch and three branches without success. We have assumed here that before the proof procedure the system S

[2]The representation by proof trees makes use of the value definiteness of availability propositions which can be justified in some way (cf. Mittelstaedt and Stachow, 1978).

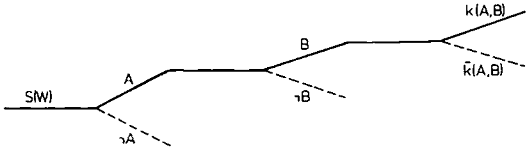

Fig. 1. Proof tree for $A \wedge B$.

was prepared such that the proposition W was true, which is denoted by $S(W)$. This initial proposition W is called here the *preparation*.

The set of material propositions can now be extended by incorporating arbitrary interactions of the connectives and availability propositions. In this way one arrives at the set $\mathfrak{S}_Q = \{\mathfrak{S}_e; \sqcap, \wedge, \vee, \to, \neg, k(\ ,\ ), \bar{k}(\ ,\ )\}$ of propositions, which is called here the Q language (Mittelstaedt, 1978; Stachow, 1981a). The proof procedures for propositions $A \in \mathfrak{S}_Q$ are summarized in the material dialogue game D_m (Mittelstaedt, 1978; Stachow, 1981a). If a proposition A can be proved in the system S with preparation W by means of a D_m dialog we write $S(W) \vdash_{D_m} A$ and call A "true in $S(W)$." If the truth of A does not depend on the particular system S, but only on its preparation W, we write $W \vdash_{D_m} A$ and call A "true with respect to W." Since the truth of a proposition $A \in \mathfrak{S}_Q$ must be demonstrated by a material dialogue, these propositions are called dialogue-definite. A material dialogue consists of a series of material proofs which are performed according to the dialogue rules by the participants of the dialogue (Mittelstaedt, 1978; Stachow, 1981a). Hence in the same way as for material propositions the dialogic proof for propositions $A \in \mathfrak{S}_e$ must be performed within the provability region R of B_U. The set $\mathfrak{S}_Q$ of propositions which are dialog-definite in R thus constitute the local Q language, which we denote by $\mathfrak{S}_Q(R)$.[3]

On the basis of the material dialogue game two kinds of equivalence relations between propositions $A \in \mathfrak{S}_Q$ can be defined which will be briefly mentioned (cf. Stachow, 1981a, p. 272). Two propositions A and B are *dialog equivalent*, denoted by $A \equiv B$, if in each dialog in which one of the two propositions occurs it may be replaced by the other one without thereby influencing the possibilities of success in the dialogue. On the other hand two propositions A and B are *value equivalent*, denoted by $A = B$, if in each dialogue in which one of these propositions is asserted, it may be replaced by the other one, such that the possibilities of success in the subdialogues about A and B are the same.

[3]By introducing convenient equivalence classes of propositions, one obtains the Lindenbaum–Tarski algebra of $\mathfrak{S}_Q$ (Stachow, 1981a). In Hilbert space formalism this algebra is realized by a subalgebra of the algebra of local observables in R (Stachow, 1981a; Schlieder, 1971).

Once the language $\mathfrak{S}_Q$ has been established, one can define a probability concept on the basis of this language (Stachow, 1978b; 1979; 1981b). For a system S with the preparation W ($W \in \mathfrak{S}_Q$) we first consider the conditional probability $p_{\langle W \rangle}(A) \in [0, 1]$ for the A outcome of an A trial, where $A \in \mathfrak{S}_Q$ is a material proposition. In the same way as the elements of the language $\mathfrak{S}_Q$, the conditional probability $p_{\langle W \rangle}(A)$ of material propositions A is related to an individual system $S(W)$ and considered as a property of its initial preparation W. However, the experimental test of a probability proposition $P_{\langle W \rangle}(A)$ must be performed by estimating the relative frequency of A outcomes of a large number of A trials in systems $S(W)$ with equivalent preparations W.[4]

The probabilities of compound propositions $A \in \mathfrak{S}_Q$ which contain a finite number of connectives can then be obtained from the proof tree for A with the initial preparation W. If the probability for a successful branch b_i in this proof tree is denoted by $p_{\langle W \rangle}(b_i(A))$ the probability for A can be expressed by

$$p_{\langle W \rangle}(A) = \sum_i p_{\langle W \rangle}(b_i(A)) \tag{1}$$

Furthermore by means of the material dialogue game one finds the relation

$$p_{\langle W \rangle}(A \sqcap B) = p_{\langle W \rangle}(A) p_{\langle W \sqcap A \rangle}(B) \tag{2}$$

The totality of recoursive formulas, which allow one to reduce the probability of a compound proposition A to the a priori probabilities of the material propositions $m_i(A)$ contained in A, can be summarized in a probability calculus for conditional probabilities (Stachow, 1978b; 1979; 1981b), the rules of which can be obtained all together from the material dialogue game. This calculus $\mathcal{P}_c$ can be found in Stachow (1981b) and will not be presented here. Some applications of this calculus will be discussed in the next section.

Finally it should be mentioned that analogous to propositions $A \in \mathfrak{S}_Q$ also probabilities $p_{\langle W \rangle}(A) \in [0, 1]$ must be measured in the provability region R of the local language considered. Hence we are dealing here more precisely with local probability propositions, the verification of which must be performed in a finite region R of space-time.

[4] The question whether probabilities can be measured only by means of relative frequencies of an ensemble of systems, or whether there is also a method of direct estimation at a single system is investigated in Koppe and Zapp (1983). (Cf. also note 6.)

4. THE MEASURING PROCESS

In this section we will briefly discuss the concept of measurement which is used implicitly in the language $\mathfrak{S}_Q$ of quantum physics. It will turn out that we are dealing here with ideal measurements of the first kind, which are of the Lüders (1951) type. According to the previous section, for a physical system S which is prepared such that $W \in \mathfrak{S}_Q$ is true, we write $S = S(W)$ and call W the preparation of S. For the material proof of another proposition A, a measuring process for A must be performed, which will be assumed to have two possible outcomes, A and $\neg A$ such that after the measuring process for the system S the propositions $W \sqcap A$ or $W \sqcap \neg A$, respectively, are true. If the conditional probabilities for A and $\neg A$ are given by $p_{\langle W \rangle}(A)$ and $p_{\langle W \rangle}(\neg A)$, respectively, in a first step (I) of the measuring process one obtains an ensemble of possible outcomes (propositions) and their respective probabilities $\Gamma(W; A) := \{p_{\langle W \rangle}(A_i), W \sqcap A_i\}_i$ which represents the observer's *knowledge* after this step (I). In a second step (II) this ensemble $\Gamma(W; A)$ can be reduced simply by reading to one of the possible components $W \sqcap A_i$, i.e., the measuring process reads (Mittelstaedt, 1976b)

$$W \xrightarrow{\text{(I)}} \Gamma(W; A) \xrightarrow{\text{(II)}} W \sqcap A_i$$

It is assumed here that these two steps of the measuring process correspond to the following description in terms of Hilbert space: If W corresponds to the projection operator P_W, $W \sqcap A$ corresponds to the component $P_A P_W P_A / \mathrm{Tr}(P_A P_W)$ of the Lüders mixture $(W; A) = \Sigma_i P_{A_i} P_W P_{A_i}$. The ensemble $\Gamma(W; A)$ corresponds to an assemblage of components of this kind and their probabilities, which is statistical equivalent to the Lüders mixture $(W; A)$. Hence the ensemble $\Gamma(W; A)$ will be called a *Lüders ensemble*. It should be noted that in step (I) of the measuring process the mentioned assemblage of state operators and not the equivalent mixture $(W; A)$ is produced. This fact can be explained by the macroscopic dimensions of the measuring instruments. Obviously there are no problems in applying the ignorance interpretation to the ensemble $\Gamma(W; A)$ (Ochs, 1981).

Here we are not dealing with Hilbert-space quantum mechanics. Instead we have rather to describe the measuring process solely in terms of the quantum language $\mathfrak{S}_Q$. In this terminology the measuring process [for A at $S(W)$] corresponds to the proof tree in Figure 2, i.e., in step (I) we obtain $W \sqcap (A \vee \neg A)$ which in step (II) is reduced to $W \sqcap A$ (or $W \sqcap \neg A$). Hence the Lüders ensemble $\Gamma(W; A)$ is adequately described by $W \sqcap (A \vee \neg A)$

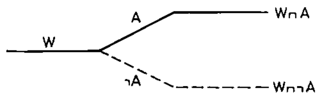

Fig. 2. Proof tree for A.

and its A component, say, by $W \sqcap A$. Consequently, the measuring process reads in terms of Q language

$$W \overset{(\mathrm{I})}{\rightarrow} W \sqcap (A \vee \neg A) \overset{(\mathrm{II})}{\rightarrow} W \sqcap A_i$$

The two branches of the proof tree have the a priori probabilities $p_{\langle W \rangle}(A)$ and $p_{\langle W \rangle}(\neg A)$, respectively, where A is considered here as an elementary proposition. For the compound proposition $A \vee \neg A$, i.e., for the sequential propositon $W \sqcap (A \vee \neg A)$, both branches are successful processes. Since according to the quantum probability calculus (Stachow, 1981b) the probability for a compound proposition is given by the sum (1) over the a priori probabilities of successful branches we have

$$p_{\langle W \rangle}(A \vee \neg A) = p_{\langle W \rangle}(A) + p_{\langle W \rangle}(\neg A) = 1 \tag{3}$$

We now consider the more complicated measuring process of the subsequent measurements of two propositions A and B. If we are interested in the outcome B, we have to investigate the successful processes for the sequence $W \sqcap (A \vee \neg A) \sqcap B$. In the proof tree shown in Figure 3, for subsequent measurements of A and B, there are two branches of success for B which correspond to the sequential propositions $A \sqcap B$ and $\neg A \sqcap B$, respectively. Hence the probability for $(A \vee \neg A) \sqcap B$ reads

$$p_{\langle W \rangle}((A \vee \neg A) \sqcap B) = p_{\langle W \rangle}(A \sqcap B) + p_{\langle W \rangle}(\neg A \sqcap B) \tag{4}$$

The probabilities of the two branches $A \sqcap B$ and $\neg A \sqcap B$ can be further

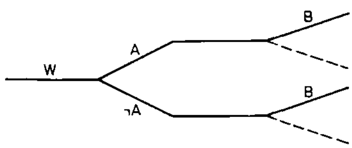

Fig. 3. Proof tree for $(A \vee \neg A) \sqcap B$.

decomposed into (cf. Stachow, 1979; 1981b)

$$p_{\langle W\rangle}(A \sqcap B) = p_{\langle W\rangle}(A) p_{\langle W \sqcap A\rangle}(B)$$

$$p_{\langle W\rangle}(\neg A \sqcap B) = p_{\langle W\rangle}(\neg A) p_{\langle W \sqcap \neg A\rangle}(B) \tag{5}$$

This measuring program can be compared with the direct measurement of B at $S(W)$. It is obvious that the sequential propositions $W \sqcap B$ and $W \sqcap (A \vee \neg A) \sqcap B$ are not dialogue equivalent, even if A and B are commensurable. Since W and A are not assumed to be commensurable, the experimental test of A may result in changes of $S(W)$ which have some influence on the result of a succeeding B test. Hence the direct test of B at $S(W)$ may lead to a different result. However, it can be shown that in spite of this difference of the propositions $W \sqcap B$ and $W \sqcap (A \vee \neg A) \sqcap B$ the probabilities for B and $(A \vee \neg A) \sqcap B$ with respect to W are equal, provided the propositions A and B are commensurable. This result, which is important for the investigations of the next section, can be demonstrated in the following way.

The probability of the elementary proposition B at $S(W)$ is given by the a priori probability $p_{\langle W\rangle}(B)$. On the other hand the probability for $(A \vee \neg A) \sqcap B$ at $S(W)$ follows from (4) and (5)

$$p_{\langle W\rangle}((A \vee \neg A) \sqcap B) = p_{\langle W\rangle}(A) p_{\langle W \sqcap A\rangle}(B) + p_{\langle W\rangle}(\neg A) p_{\langle W \sqcap \neg A\rangle}(B)$$

and is illustrated by the proof tree in Figure 3. If A and B are commensurable, i.e., $\vdash k(A, B)$ then in every single branch of this proof tree the temporal order in which the proofs of A and B are performed is irrelevant for the relative frequency of this branch. Replacing the two branches of success $W \sqcap (\neg A \sqcap B)$ and $W \sqcap (A \sqcap B)$ by $W \sqcap (B \sqcap A)$ and $W \sqcap (B \sqcap \neg A)$ one gets the new proof tree shown in Figure 4, with probabilities

$$p_{\langle W\rangle}(\neg A \sqcap B) = p_{\langle W\rangle}(B \sqcap \neg A) = p_{\langle W\rangle}(B) p_{\langle W \sqcap B\rangle}(\neg A)$$

$$p_{\langle W\rangle}(A \sqcap B) = p_{\langle W\rangle}(B \sqcap A) = p_{\langle W\rangle}(B) p_{\langle W \sqcap B\rangle}(A)$$

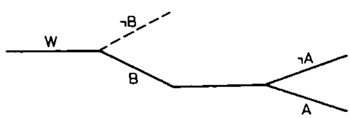

Fig. 4. Proof tree for $B \sqcap (A \vee \neg A)$.

Hence we obtain

$$\begin{aligned} p_{\langle W\rangle}((A\vee\neg A)\sqcap B) &= p_{\langle W\rangle}(B\sqcap A)+p_{\langle W\rangle}(B\sqcap\neg A) \\ &= p_{\langle W\rangle}(B)\big(p_{\langle W\sqcap B\rangle}(A)+p_{\langle W\sqcap B\rangle}(\neg A)\big) \\ &= p_{\langle W\rangle}(B)p_{\langle W\sqcap B\rangle}(A\vee\neg A)=p_{\langle W\rangle}(B) \qquad (6) \end{aligned}$$

which is the desired result.

According to the probability calculus mentioned (Stachow, 1981b), the left-hand side of (6) can be written as

$$p_{\langle W\rangle}\{(A\vee\neg A)\sqcap B\}=p_{\langle W\sqcap(A\vee\neg A)\rangle}(B)/p_{\langle W\rangle}(A\vee\neg A)$$

Since $p_{\langle W\rangle}(A\vee\neg A)=1$, we finally arrive at the following result:

L Theorem. If A and B are commensurable, we have

$$p_{\langle W\sqcap(A\vee\neg A)\rangle}(B)=p_{\langle W\rangle}(B) \qquad (7)$$

In this formulation the theorem states that the probability of B with respect to preparation W is equal to the probability of B with respect to the Lüders ensemble $\Gamma(W; A)$, provided A and B are commensurable.

In Hilbert space quantum theory the physical content of the L theorem (7) is a well-known result, which was first demonstrated by Lüders (1951). However, for the present investigation it is important that the theorem can be proved solely by means of Q language and the corresponding probability calculus.

5. SPACE-TIME REGIONS

We consider an individual physical system S with the preparation W, and properties $A, B,\ldots$ which can be tested at $S=S(W)$. The preparation W as well as other measurable propositions $A, B,\ldots$ are assumed here to be time independent, i.e., in terms of quantum theory we use the Heisenberg representation for the elements of the language $\mathfrak{S}_Q$. In the framework of relativistic quantum theory, these Heisenberg propositions are first of all related to the entire four-dimensional space-time $\mathfrak{M}$. On the other hand the measuring processes for propositions $A, B,\ldots\in\mathfrak{S}_Q$ are performed in finite regions of space-time and hence the validity of a given proposition A must be restricted to a certain validity region $M=R_v(A)$ of space-time $\mathfrak{M}$. In order to indicate the validity region of a proposition A in the following we write $A(M)$ with $M = R_v(A)$.

The historical development of a system S is thus determined by the succession of measuring processes for several propositions which are performed at the system. The preparation W is the result of a physical process which has been performed in the early part of the system. Its validity region will be denoted by M_0. If we consider the measurement of the proposition A, which is performed at point x^* in space-time, and which has the outcome A, the history of S reads

$$\{W(M_0), W \sqcap A(M_1)\}$$

where the validity regions M_0 and M_1 are such that $M_0 \cup M_1 = \mathfrak{M}$ and $M_0 \cap M_1 = \varnothing$. This description of the history of S corresponds to the A branch of the proof tree shown in Figure 5, where the branching point marks the beginning of a new section in the history of the system. The vertical line σ_W indicates the hypersurface which separates the regions M_0 and M_1 before and after the measurement.

In Newtonian space-time the hypersurface σ_W is given by $x_0 = x_0^*$ (with $x_0 = ct$) and thus the validity regions are

$$M_0 = \{x: x_0 \leqslant x_0^*\}, \qquad M_1 = \{x: x_0 > x_0^*\}$$

Since this separation is Galilei invariant, it is relevant for all observers B_U in x^* irrespective of their velocity. However, in Minkowskian space-time of special relativity this definition of σ_W can no longer be used, since σ_W is not invariant under homogeneous Lorentz transformations and thus the validity regions would depend on the velocity of the respective observer in x^*. Instead one has to define here a Lorentz-invariant hypersurface, which separates M_0 and M_1 in a convenient way. There are two possible choices for a Lorentz-invariant σ_W, the forward light cone $L^{(+)}(x^*)$ in x^* and the backward light cone $L^{(-)}(x^*)$, respectively. Here we define σ_W by the backward light cone $L^{(-)}(x^*)$ since otherwise long-range correlations between the measuring event x^* and points y^* with spacelike distances would be excluded. The existence of nonlocal correlations in EPR-like situations,

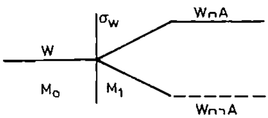

Fig. 5. Proof tree for A and validity regions.

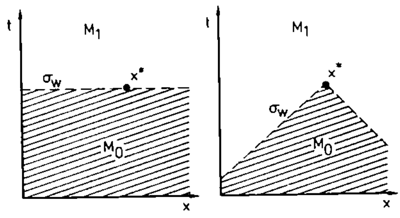

Fig. 6. Validity regions in Newtonian and Minkowskian space-time.

however, seems to be a very important feature of quantum physics even if presently there is not yet direct experimental evidence for it. Hence the possibility of such correlations should in any case be incorporated into the scientific language. For this reason, we define the relativistic hypersurface σ_W by $x_0 = x_0^* - |x_k - x_k^*|$. (Figure 6).

If once this definition of the validity regions is accepted, we can investigate a more complicated measuring program which consists of two subsequent measurements of A and B and space-time points x^* and y^*. "Subsequent" means in the relativistic context that x^* and y^* are points on the timelike world line of the observer B_U with $x_0^* < y_0^*$. The history of S is then described by a branch in the proof tree for A and B, shown in Figure 7, e.g., by $W \sqcap A \sqcap B$ and reads

$$\{W(M_0), W \sqcap A(M_1), (W \sqcap A) \sqcap B(M_2)\}$$

Since the measuring processes are performed at the points x^* and y^*, the hypersurfaces σ_W and σ_A which separate the regions M_0, M_1, M_2 are given by $L^{(-)}(x^*)$ and $L^{(-)}(y^*)$, respectively. The corresponding validity regions M_0, M_1, and M_2 are shown in Figure 8 and are given by

$$M_0 = J^{(-)}(x^*), \qquad M_1 = J^{(-)}(y^*) \cap \overline{J^{(-)}(x^*)}, \qquad M_2 = \overline{J^{(-)}(y^*)}$$

where we have denoted by $J^{(-)}(x)$ the causal past of x, i.e., the union of the backward light cone $L^{(-)}(x)$ and its interior, and by $\overline{J^{(-)}(x)}$ the comple-

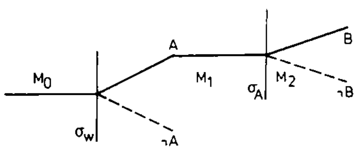

Fig. 7. Proof tree for $A \sqcap B$ and validity regions.

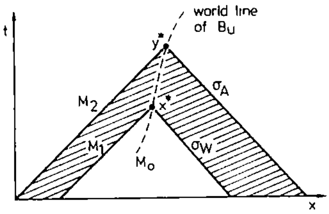

Fig. 8. Validity regions for points x^* and y^* with timelike distance.

ment of this set. Obviously they fulfill the relations $M_0 \cup M_1 \cup M_2 = \mathfrak{M}$, $M_i \cap M_k = \varnothing$ for $i \neq k$.

A division of the Minkowskian space into distinct regions M_i which are determined by measuring processes, will be called a measuring chart or M chart of $\mathfrak{M}$. The definition of M charts, which is given here for two event points x^* and y^* with timelike distances, can easily be extended to more than two events and to events with spacelike distances. An example with spacelike points x^* and y^* is shown in Figure 9.

The description of the history of a system $S(W)$ is more complicated if the measurements of A and B are performed at points x^* and y^* with a spacelike distance. In this case it is no longer possible to think of an observer B_U, who describes the history of $S(W)$ and who performs the measurements in x^* and y^*. Instead we have clearly to distinguish between the measuring instruments at x^* and y^*, which completely perform the measurements of A and B, respectively, and the observer B_U, who gives a description of the history of $S(W)$ on the basis of the results of these measurements. It is obvious that the information about these measuring results must be transmitted from x^* and y^* to the observer's space-time point $\xi(B_U)$ by convenient signals. Since all kinds of classical signals, which can be received by B_U, are restricted by the principle of Einstein causality, the space-time point $\xi(B_U)$ of the observer becomes important for the description of the history of the system $S(W)$.

We assume that there is a well-defined measuring program such that proposition A is measured at x^* and B at y^*. The measurements are completely performed by the instruments, i.e., both steps of the measuring process are carried through and the result which is obtained in step (II) by reading is registered in some way. Hence a distant observer B_U may either be informed about the final result of the measurement, or his knowledge about the outcome is restricted merely by subjective ignorance. Here we assume that the observer obtains all information which (in the relativistic sense) is available to him.

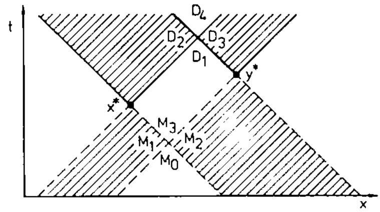

Fig. 9. Validity regions and domains D_i of information for points x^* and y^* with spacelike distance.

According to this measuring program, there are four sections M_0, M_1, M_2, M_3 of space-time, which determine the validity regions of the objectively decided propositions A and B (Figure 9). Here we have

$$M_0 = J^{(-)}(x^*) \cap J^{(-)}(y^*), \qquad M_1 = J^{(-)}(x^*) \cap \overline{J^{(-)}(y^*)}$$

$$M_2 = \overline{J^{(-)}(x^*)} \cap J^{(-)}(y^*), \qquad M_3 = \overline{J^{(-)}(x^*)} \cap \overline{J^{(-)}(y^*)}$$

where the complement of $J^{(-)}$ is denoted by $\overline{J^{(-)}}$. However, the description of the history of the system S, which is given by the observer B_U, depends not only on the objective situation (in the quantum physical sense) which is described by the given M chart, but also on the information about this objective situation, which is available (in the relativistic sense) to the observer B_U at the space-time point $\xi(B_U)$. In this sense the history of the system S depends on the space-time position of the observer B_U.

If once the points x^* and y^* are given and if, say, $x_0^* < y_0^*$, then we can distinguish four distinct domains D_1, D_2, D_3, D_4 of the Minkowski space $\mathfrak{M}$, which correspond to different possibilities of information, Figure 9. In the present example, we have

$$D_1 = \overline{J^{(+)}(x^*)} \cap \overline{J^{(+)}(y^*)}, \qquad D_2 = J^{(+)}(x^*) \cap \overline{J^{(+)}(y^*)}$$

$$D_3 = \overline{J^{(+)}(x^*)} \cap J^{(+)}(y^*), \qquad D_4 = J^{(+)}(x^*) \cap J^{(+)}(y^*)$$

where we have denoted by $J^{(+)}(x^*)$ the causal future of x^*, i.e., the union of the forward light cone $L^{(+)}(x^*)$ and its interior. For the description of the history of the system $S(W)$ it is important in which of these domains D_i the observer's space-time point $\xi(B_U)$ is located. Even if the objectively decided result of a measurement of A, say, is not known to B_U, he knows that the measuring process has been completely performed at x^* and that a Lüders ensemble $W \sqcap (A \vee \neg A)$ has been generated.

Hence in the different domains D_i the observer will give the following alternate descriptions of the history of the system S in the region M_3:

$$\xi(B_u) \in D_1: \{W \sqcap (A \vee \neg A) \sqcap (B \vee \neg B\}$$

$$D_2: \{W \sqcap A_k \sqcap (B \vee \neg B)\}$$

$$D_3: \{W \sqcap (A \vee \neg A) \sqcap B_l\}$$

$$D_4: \{W \sqcap A_k \sqcap B_l\}$$

Obviously for the other regions M_1 and M_2 similar sets of alternate descriptions can be given.

6. THE CONSISTENCY OF THE LANGUAGE

An observer B_U at the space-time point $\xi(B_u)$ describes the system S and its history by means of the quantum language $\mathfrak{S}_{RQ}$ and a coordinate system K_B, the origin of which is at rest relative to B_U. The restrictions which must be imposed on this quantum language $\mathfrak{S}_{RQ}$ due to relativity are taken into account by the validity regions M_i of propositions in Minkowski space and by considering the respective domain of information D_i of the observer's position ξ in space-time. So far Lorentz invariance and Einstein causality are incorporated into the relativistic quantum language $\mathfrak{S}_{RQ}$. However, it is not yet clear, whether this language is consistent in the following sense. The history of a system S consists of a sequence of facts, which are objective in the sense of quantum physics. It may happen that the observer's knowledge of these facts is incomplete and depends on his position and his particular Lorentz frame. However, it must be required that the descriptions of S, which are given by different observers B, B',... do not contradict each other. It will be shown that this requirement corresponds to an additional consistency postulate, the C postulate, which must be imposed onto $\mathfrak{S}_{RQ}$.

There is still another possible inconsistency of a more subtle kind which one must be prepared for. In the present approach we have presupposed the principle of Einstein causality for all kinds of classical signals. However, the M charts of Minkowski space are defined such that nonlocal correlations between spacelike points, which are known from the EPR thought experiment, are not excluded. Hence it could happen that by means of these correlations superluminal signals can be constructed which would violate the principle of Einstein causality at least for quantum physical signals. Whether the nonlocal EPR correlations really exist is still an open question (d'Espagnat, 1979). However, it can be shown that irrespective of the existence of these correlations, signals which violate Einstein causality cannot be constructed, provided the above-mentioned consistency postulate is fulfilled.

We consider again the above-mentioned measuring program with measurement of A and B at spacelike points x^* and y^*. Let the coordinate system $K_B(x)$ be at rest relative to the observer B_U, whose position ξ will be assumed to be located in D_4. If the realistic history of the system S has an absolute meaning, as was discussed above, the change of the Lorentz frame of B_U by a homogeneous Lorentz transformation should not have any influence on this history. If the Lorentz group Λ is represented in Minkowskian space by transformations $x^\mu \to x^{\mu'} = \Lambda^\mu_\nu x^\nu$, we assume that Λ is represented in the language $\mathfrak{S}_{RQ}$ by a group of automorphisms α_Λ: $\mathfrak{S}_{RQ} \to \mathfrak{S}_{RQ}$ such that proposition $A' = \alpha_\Lambda(A)$ with $A \in \mathfrak{S}_{RQ}$ is the transformed proposition A. In the algebraic representation of quantum logic this group of automorphisms is well known (Gudder, 1971). The generalization of these investigations to the present approach seems to be possible.

Let Λ^μ_ν be a Lorentz transformation such that the temporal order $x_0^* < y_0^*$ is changed into $x_0^{*\prime} > y_0^{*\prime}$. The history h of S in the region M_3, say, is transformed under Λ into

$$h'(M_3): (W' \sqcap A'_k) \sqcap B'_l \tag{8}$$

whereas the history of S in the transformed region M'_3 is given by

$$h(M'_3): (W' \sqcap B'_l) \sqcap A'_k \tag{9}$$

If the history has the required absolute meaning, the two propositions (8) and (9) must be equivalent, i.e.,

$$W \sqcap (A_k \sqcap B_l) = W \sqcap (B_l \sqcap A_k) \tag{10}$$

and this for all preparations W and all indices k, l. (We have omitted here the prime, since it indicates merely that the propositions are formulated in the language $\mathfrak{S}'_{RQ}$.)

The equivalence sign " = " in (10) means value equivalence with respect to truth values (Stachow, 1981b). Since the validity of this equivalence is demanded here for all W, we have

$$A_k \sqcap B_l = B_l \sqcap A_k \tag{11}$$

for all k and l.

The four equivalence relations (11) imply the commensurability of A and B. In order to demonstrate this important result, we consider the first

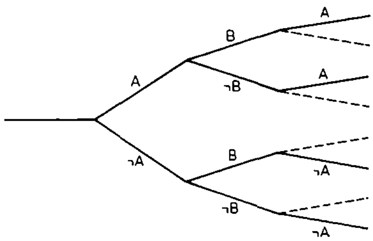

Fig. 10. Proof tree for $K(A, B)$.

two steps of the proof tree for the proposition $k(A, B)$ (Figure 10). This proof tree consists of four successful branches and of four branches without success (dotted lines). According to the equivalence (11) these branches can equivalently be replaced by

$$A \sqcap B \sqcap \neg A = A \sqcap \neg A \sqcap B$$
$$A \sqcap \neg B \sqcap \neg A = A \sqcap \neg A \sqcap \neg B$$
$$\neg A \sqcap B \sqcap A = \neg A \sqcap A \sqcap B$$
$$\neg A \sqcap \neg B \sqcap A = \neg A \sqcap A \sqcap B$$

Hence each of these reformulated branches contains $A \sqcap \neg A$ or $\neg A \sqcap A$ and will thus never appear in a proof process. Since this argument can immediately be iterated to $k(A, B)$ proof trees of arbitrary length, we find that A and B are commensurable, i.e., $\vdash k(A, B)$, if the equivalence (11) is fulfilled.

Summarizing these investigations we can express the consistency postulate for the language $\mathfrak{S}_{RQ}$ in the following way:[5]

C-postulate. If propositions A and B are provable at points x^* and y^* of $\mathfrak{M}$ with a spacelike distance, then A and B are commensurable.

In relativistic Hilbert space quantum theory, this C postulate corresponds to the well-known *locality condition* (Streater and Wightman, 1964) which is generally assumed to be valid in relativistic quantum field theory. In the present approach it plays the role of a consistency postulate for the relativistic quantum language $\mathfrak{S}_{RQ}$. It is important to note that the derivation of the C postulate from the consistency requirement for the relativistic quantum language has been performed here merely by means of the Q language and without any recourse to the Hilbert space quantum theory.

[5]Commensurability propositions $k(A, B)$ are defined first of all solely for propositions A, B with the same provability region (Section 3). The C postulate gives an extension of this definition for provability regions with a spacelike distance, and is thus always compatible with the original definition of $k(A, B)$.

We can now combine this C postulate with the L theorem of Section 4. In this way we obtain the following result:

L-C Theorem. If propositions A and B are provable at points x^* and y^*, respectively, with spacelike distance, then the probabilities of B with respect to W and $W \sqcap (A \wedge \neg A)$ are equal, i.e.,

$$p_{\langle W \rangle}(B) = p_{\langle W \sqcap (A \wedge \neg A) \rangle}(B)$$

This theorem means that the probability for the outcome B of a B trial at the point y^* with respect to the preparation W is the same as the probability with respect to the Lüders ensemble $\Gamma(W; A)$ which is generated by an A test at $S(W)$, provided the point x^* of the A trial has spacelike distance to y^*. Consequently the probability of a B test at the point y^* does not depend on whether at the spacelike point x^* an A trial has been carried out or not. For this reason it is impossible from the point x^* to the spacelike point y^* to transmit a superluminal signal, which uses the alternative (A test, no A test) and the effect of which consists in a change of the B probability.

Since probabilities as well as the language $\mathfrak{S}_{RQ}$ are related throughout this paper to a single system, a change of the B probability could be received as a message at a single system $S(W)$.[6] Hence the impossibility of such probability signals means that the quantum physical nonlocal correlations between spacelike points (if they exist) cannot be used in order to violate the principle of Einstein causality. The above-mentioned question, whether the C postulate also guarantees the validity of Einstein causality at the quantum level, can thus be answered affirmatively.

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[6]Probabilities $p_{\langle W \rangle}(A)$ of elementary propositions A are considered here as measurable properties of the system $S(W)$. In Hilbert space quantum physics with $W \doteq P_\varphi$ and $A \doteq P_A$ the probability $p_\varphi(P_A)$ is obtained by measuring the observable with the operator $P_\varphi P_A P_\varphi$ at the system S with the state φ.

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